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Broadcasting Problem in a Specific Class of Graphs

By: Saber Gholami

Supervisor: Professor Hovhannes Harutyunyan

Concordia University,

Department of Computer Science and Software Engineering

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- Growth of using computer networks,
- Great attention to all major problems in this area,
- Information dissemination,
- Broadcasting:
 - ◇ Process of distributing a message starting from a single node (*originator*) to all other nodes of the network using the network's links.



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- The network: $G = (V, E)$, originator $u \in V$.



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- The network: $G = (V, E)$, originator $u \in V$.
- $b(u, G)$: minimum time required to finish the broadcasting originating from u .
- $b(G) = \max\{b(u, G) | u \in V(G)\}$
 - ◇ For any graph: $b(G) \geq \lceil \log n \rceil$



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- The network: $G = (V, E)$, originator $u \in V$.
- $b(u, G)$: minimum time required to finish the broadcasting originating from u .
- $b(G) = \max\{b(u, G) | u \in V(G)\}$
 - ◇ For any graph: $b(G) \geq \lceil \log n \rceil$
- Two major problems in this area:
 - ◇ **Broadcast time problem,**
 - ◇ Network design.



Literature Review - Broadcast time problem

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- NP-Complete in arbitrary graphs [22],
 - ◇ Remains NP-Complete even in more restricted families of graphs such as planar and decomposable graphs [17].



Literature Review - Broadcast time problem

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- NP-Complete in arbitrary graphs [22],
 - ◇ Remains NP-Complete even in more restricted families of graphs such as planar and decomposable graphs [17].
- Exact solutions:
 - ◇ Trees [22],
 - ◇ Unicyclic graph [11],
 - ◇ Necklace graph [9]
 - ◇ Tree of cycles, Tree of cliques [19]



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- Approximation algorithms:
 - ◇ $(\sqrt{|V|})$ -additive approximation [18],
 - ◇ $(\frac{\log^2 |V|}{\log \log |V|})$ -approximation algorithm [20],
 - ◇ $(\frac{\log k}{\log \log k})$ -approximation algorithm for multicasting [5]
 - Or a $(\frac{\log |V|}{\log \log |V|})$ -approximation solution for broadcasting [5]



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- Approximation algorithms:
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 - ◇ $(\frac{\log^2 |V|}{\log \log |V|})$ -approximation algorithm [20],
 - ◇ $(\frac{\log k}{\log \log k})$ -approximation algorithm for multicasting [5]
 - Or a $(\frac{\log |V|}{\log \log |V|})$ -approximation solution for broadcasting [5]
- Inapproximability of the problem:
 - ◇ broadcast time could not be approximated within a factor of $\frac{57}{56} - \epsilon$ for an arbitrary graph [21]
 - ◇ NP-Hard to approximate the broadcast problem within a ratio of $(3 - \epsilon)$, for any $\epsilon > 0$ [4].



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- Heuristics:
 - ◇ Round Heuristic: $O(R|V||E| \log |V|)$ [1]
 - ◇ Tree Based Algorithm: $O(R|E|)$ [12]
 - ◇ Random heuristic: $O(|E|)$ [14]
 - Semi-random version: $O(|E|)$ [13]
 - ◇ Based on Genetic Algorithm [16]



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- A Hypercube of Trees HT_k :
 - ◇ A hypercube of dimension $k + 1$
 - ◇ 2^k arbitrary trees.
- Very useful structure:
 - ◇ Distributing data [15],
 - ◇ Simultaneous exchange of packets between processors [2],
 - ◇ ...
- Current upperbound:
($2 - \varepsilon$)-approximation [3]

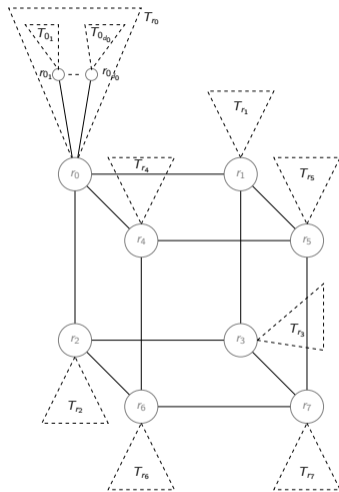


Figure: HT_3 , A hypercube of trees with dimension 3



HT_k - Proposed Heuristic

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- Instead of finding $b(u, HT_k)$, solve this:
 - ◇ $b(u, HT_k) \leq \tau$?



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- Instead of finding $b(u, HT_k)$, solve this:

$$\diamond b(u, HT_k) \leq \tau?$$

- Already know the upper bound and lower bound:

$$\underbrace{\max \left\{ k, \max_{0 \leq i \leq 2^k - 1} \{b(r_i, T_i)\} \right\}}_{lb} \leq b(u, HT_k) \leq k + \underbrace{\max_{0 \leq i \leq 2^k - 1} \{b(r_i, T_i)\}}_{ub}$$



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- Instead of finding $b(u, HT_k)$, solve this:

- ◇ $b(u, HT_k) \leq \tau?$

- Already know the upper bound and lower bound:

$$\underbrace{\max \left\{ k, \max_{0 \leq i \leq 2^k - 1} \{b(r_i, T_i)\} \right\}}_{lb} \leq b(u, HT_k) \leq k + \underbrace{\max_{0 \leq i \leq 2^k - 1} \{b(r_i, T_i)\}}_{ub}$$

- Do a binary search on this range (*MBS*).
 - ◇ Invoke the main heuristic (*br*) within this function.



HT_k - Proposed Heuristic - MBS

Algorithm 1 The Modified Binary Search $MBS(G, r_i, lb, ub)$

input : $HT_k = (V, E)$, originator r_i , lower bound lb , and the upper bound ub

output: An improved broadcast time for $b(r_i, HT_k)$ denoted by b

$b = ub$

if $ub < lb$ **then**

 | **return** *FALSE*

end

$mid = lb + \lfloor \frac{ub-lb}{2} \rfloor$

if $br(HT_k, r_i, mid)$ **then**

 | **update** $b = mid$

 | **return** $MBS(G, r_i, lb, mid - 1)$

else

 | **return** $MBS(G, r_i, mid + 1, ub)$

end

return b

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- Considering a root vertex r_i at time t :
 - ◇ $b(r_i, T_i) > \tau - t$:
 - return FALSE!
 - ◇ $b(r_i, T_i) = \tau - t$:
 - r_i broadcasts in its tree T_i .
 - ◇ $b(r_i, T_i) < \tau - t$:
 - r_i must contribute in the Hypercube...

Algorithm 2 Broadcasting heuristic $br(G, r_i, \tau)$

Input: $HT_k = (V, E)$, originator r_i , a candidate time τ .

Output: TRUE if a broadcast scheme is found, FALSE otherwise.

Initialize $V_I = \{r_i\}$, $V_U = V_{H_k} - V_I$, $covered(u \in V_U) = FALSE$,
 $covered(r_i) = TRUE$, $rem_{r_i} = NULL$

for $0 \leq t \leq \tau - 1$ do

 for $u \in V_U$ do

 | update $rem_u = \tau - t - b(u, T_u) - dist(u, V_I)$

 end

 for $r_i \in V_I$ do

 if $b(r_i, T_i) < \tau - t$ then

 if $busy(r_i)$ then

 | r_i informs v following its path, $V_I = V_I \cup \{v\}$, $V_U = V_U - \{v\}$;

 | continue;

 end

 if r_i is stuck then

 | continue;

 end

 Choose r_j with the minimum value of rem_{r_j} from Equation (3.5)

 in a way that there is at least one valid path from r_i to r_j ;

 Denote the valid paths by $VP = \{p_1, \dots, p_r\}$;

$MC = \{p \in VP | c_p = \min_{p' \in VP} c_{p'}\}$;

$selected = \{p \in MC | d_p = \min_{p' \in MC} d_{p'}\}$;

 if $|selected| > 1$ then

 | Select one randomly;

 end

 For all mid-vertices on the selected path, set $covered(mid-ver) =$

$TRUE$, $busy(mid-ver) = TRUE$;

r_i informs v which is the first vertex on the selected path;

$V_I = V_I \cup \{v\}$, $V_U = V_U - \{v\}$;

 else

 if $b(r_i, T_i) = \tau - t$ then

 | Follow the broadcast scheme in trees;

 | Update $b(r_i, T_i)$ with regard to a sub tree T_i , which has been

 | informed;

 else

 | return FALSE

 end

 end

 end

end

return TRUE

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HT_k - Proposed Heuristic - br - cont.

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- We want to send a message from a root vertex r_i to another root vertex r_j
- Two questions must be answered:
 - How to choose r_j ?
 - ◇ The one with the minimum value of rem_{r_j} :
$$rem_{r_j} = \tau - t - b(r_j, T_j) - dist(r_j, V_l)$$



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- We want to send a message from a root vertex r_i to another root vertex r_j
- Two questions must be answered:
 - How to choose r_j ?
 - ◇ The one with the minimum value of rem_{r_j} :
$$rem_{r_j} = \tau - t - b(r_j, T_j) - dist(r_j, V_l)$$
 - How to choose a path between r_i and r_j ?
 - ◇ A path P with the minimum value of c_P and d_P :
$$c_P = \min_{r_m \in P} \{rem_{r_m}\}$$

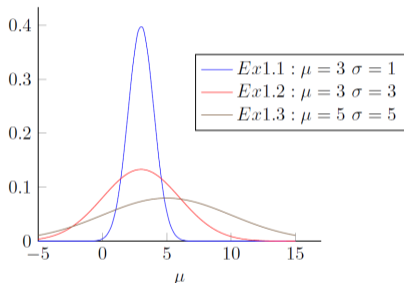
$$d_P = \sum_{r_m \in P} rem_{r_m}$$
 - ◇ More "critical" vertices on such a path!



HT_k - Evaluation - Setup

- Generate 1000 random HT_k for each k .

◇ Following Gaussian distribution $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$



- In terms of:
 - ◇ *success rate*: How many times we performed better than [3]?
 - ◇ *gain*: How much better?
 - ◇ $|V|$

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HT_k - Evaluation - Ex 1.1

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Table: Numerical Results for Ex1.1: $\mu = 3, \sigma = 1$

k	average $ V $	success rate	average gain
3	115.86	29.79%	7.55%
4	233.55	28.49%	5.84%
5	466.09	21.3%	3.71%
6	932.49	5.7%	0.85%
7	1862.98	1.3%	0.16%
8	3729.59	0.2%	0.02%



HT_k - Evaluation - Ex 1.2

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Table: Numerical Results for Ex1.2: $\mu = 3, \sigma = 3$

k	average $ V $	success rate	average gain
3	614.28	73.5%	24.45%
4	1189.54	78.6%	23.31%
5	2481.77	74.5%	19.81%
6	5008.22	69.2%	15.55%
7	10003.63	63.2%	12.5%
8	19927.99	54.7%	9.76%



HT_k - Evaluation - Ex 1.3

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Table: Numerical Results for Ex1.3: $\mu = 5, \sigma = 5$

k	average $ V $	success rate	average gain
3	185255.48	81.20%	28.65%
4	280823.85	86.00%	28.82%
5	704372.23	88.10%	27.35%
6	1313690.28	89.30%	26.70%
7	3532669.06	90.50%	25.85%
8	5245921.21	90.10%	24.10%



HT_k - Evaluation - Ex 2.1

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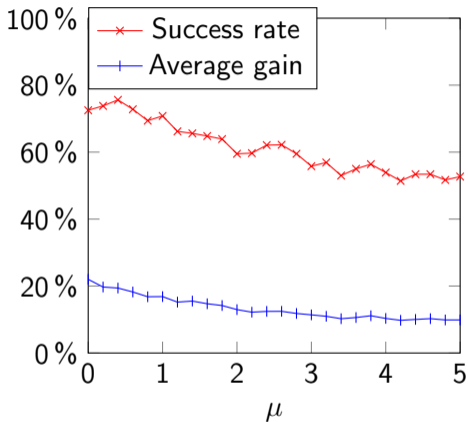


Figure: Ex2.1: $k = 5, \sigma = 2$



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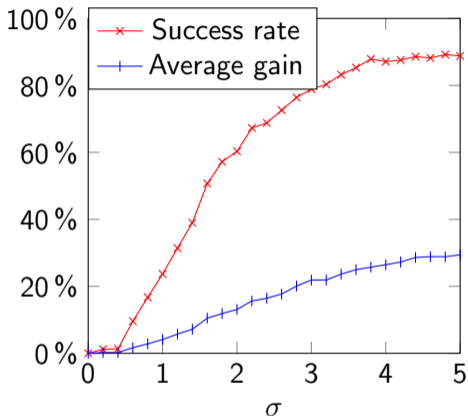


Figure: Ex2.2: $k = 5, \mu = 2$



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- We proposed a heuristic for broadcasting in a hypercube of trees.
- Theoretically:
 - ◇ 2-approximation
- Practically:
 - ◇ outperform the best-known algorithm for the same problem in up to 90% of the experiments while speeding up the process up to 30%



Fully Connected Trees

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- A Fully Connected Tree FCT_n :
 - ◇ A Clique of size $n +$
 - ◇ n arbitrary trees.
- Previous result: A $O(|V| \log |V|)$ algorithm [8]

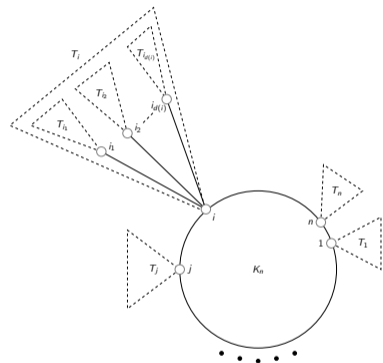


Figure: A Fully Connected Tree FCT_n



FCT_n - Broadcast Algorithm for Root Vertices

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- Instead of finding $b(i, FCT_n)$, solve this:
 - ◇ $b(i, FCT_n) \leq \tau$?



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- Instead of finding $b(i, FCT_n)$, solve this:

$$\diamond b(i, FCT_n) \leq \tau?$$

- **Lemma:**

$$\diamond \underbrace{\max \{ \lceil \log n \rceil, \max \{ b(i, T_i) \} \}}_{lb} \leq b(i, FCT_n) \leq \underbrace{\lceil \log n \rceil + \max \{ b(i, T_i) \}}_{ub}$$



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- Instead of finding $b(i, FCT_n)$, solve this:

- ◇ $b(i, FCT_n) \leq \tau?$

- **Lemma:**

- ◇ $\underbrace{\max\{\lceil \log n \rceil, \max\{b(i, T_i)\}\}}_{lb} \leq b(i, FCT_n) \leq \underbrace{\lceil \log n \rceil + \max\{b(i, T_i)\}}_{ub}$

- Do a binary search on this range (*MBS*).
 - ◇ Invoke the main algorithm (BR_τ) within this function.



FCT_n - Broadcast Algorithm for Root Vertices - cont.

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- Considering a root vertex i at time t :
 - ◇ $b(i, T_i) > \tau - t$:
 - return FALSE!
 - ◇ $b(i, T_i) = \tau - t$:
 - i broadcasts in its tree T_i .
 - ◇ $b(i, T_i) < \tau - t$:
 - i must contribute in the Clique...

Algorithm 3 The broadcast algorithm $BR_r(FCT_n, i, \tau)$

Input: $FCT_n = (V, E)$, originator i , candidate broadcast time τ

Output: FALSE if τ cannot be the broadcast time, TRUE if broadcasting can be accomplished in at most τ time units.

Initialize: the labels $w(i, t)$ and m_{ij} for all root vertices;

Initialize: $V_I = \{i\}$, $V_U = V \setminus V_I$, $l_i = \text{NULL}$;

foreach t such that $0 \leq t \leq \tau - 1$ do

 foreach $v \in V_U$ do

 if v is a root vertex then

 update l_v as follows: $l_v = \tau - t - w(v, t) - 1$;

 end

 end

 foreach $v \in V_I$ do

 if v is a root vertex then

 if $w(v, t) < \tau - t$ then

v informs vertex j at time t such that j has the smallest value of l_α in V_U ;

$l_j = \text{NULL}$, $V_I = V_I \cup \{j\}$, $V_U = V_U \setminus \{j\}$;

 else

 if $w(v, t) = \tau - t$ then

v informs one of its children which has the highest value of m_{vj} in the tree rooted at T_v , $1 \leq j \leq d(v)$;

$m_{vj} = \text{NULL}$, $V_I = V_I \cup \{v_j\}$, $V_U = V_U \setminus \{v_j\}$;

 update $w(v, t) = \max_{1 \leq k \leq d(v)} \{k + m_{vk}\}$;

 else

 return FALSE

 end

 end

 end

 else

v informs a tree vertex u in the uninformed sub-tree rooted at v

 based on the well-known broadcasting algorithm in trees;

$V_I = V_I \cup \{u\}$, $V_U = V_U \setminus \{u\}$

 end

end

end

return TRUE



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- We want to send a message from a root vertex i to another root vertex j
- Two questions must be answered:
 - How to choose j ?

- ◊ The one with the minimum value of l_j :

$$l_j = \tau - t - w(j, t) - 1,$$

$$w(i, t) = \max_{1 \leq j \leq d(i)} (j + m_{ij}),$$

$$m_{ij} = b(ij, T_{ij}).$$



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- We want to send a message from a root vertex i to another root vertex j
- Two questions must be answered:
 - How to choose j ?
 - ◇ The one with the minimum value of l_j :
$$l_j = \tau - t - w(j, t) - 1,$$
$$w(i, t) = \max_{1 \leq j \leq d(i)} (j + m_{ij}),$$
$$m_{ij} = b(ij, T_{ij}).$$
 - How to choose a path between r_i and r_j ?
 - ◇ Easy! They are connected by an edge (i, j) .



FCT_n - Proof of Correctness

Theorem 1

If Algorithm $BR_\tau(FCT_n, i, \tau)$ outputs TRUE, then $b(i, FCT_n) \leq \tau$.

Theorem 2

if Algorithm $BR_\tau(FCT_n, i, \tau)$ returns FALSE, then $b(i, FCT_n) > \tau$.

- **Lemma:** If under broadcast scheme S_τ , a root vertex i informs a tree vertex i_j at time t_1 , then it is necessary for any other scheme S for broadcasting in the same graph to inform i_j by the time t' where $t' \leq t_1$.
- **Lemma:** Assume $b(i, FCT_n) = \tau$. Let S_{opt} be an optimum broadcast scheme different than S_τ . Then, at any time t , $|V_t(S_\tau)| \geq |V_t(S_{opt})|$.
 - ◇ $V_t(S)$: The set of informed root vertices by the time t under the broadcast scheme S .

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FCT_n - Time Complexity

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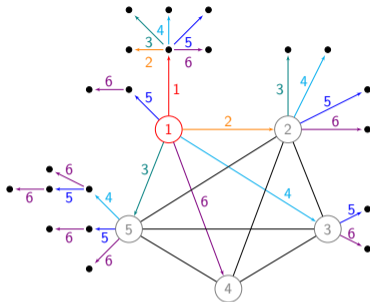
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- $O(\text{Algorithm}) \times O(\log(ub - lb))$
 - ◇ The Algorithm is linear $O(|V|)$,
 - ◇
$$ub - lb = \lceil \log n \rceil + \max\{b(i, T_i)\} - \max\{\lceil \log n \rceil, \max\{b(i, T_i)\}\} \rightarrow$$
$$ub - lb = \min\{\lceil \log n \rceil, \max\{b(i, T_i)\}\} \rightarrow$$
$$ub - lb \leq \lceil \log n \rceil. \tag{1}$$
- Complexity: $O(|V| \log \log n)$



FCT_n - Example



Labels after $t = 1$

root vertex i	$w(i, t)$	m_{i_1}	m_{i_2}	m_{i_3}	m_{i_4}	l_i
1	6 2	5 -	1	-	-	-
2	4	0	0	0	0	1 0
3	2	0	0	-	-	3 2
4	0	-	-	-	-	5 4
5	3	2	1	0	-	2 1

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FCT_k - Broadcast Algorithm for Tree Vertices

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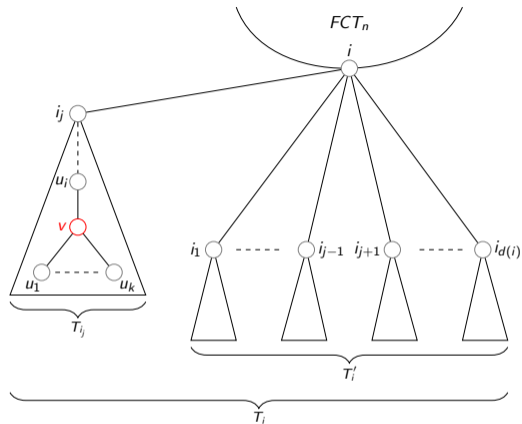
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Theorem 3

This Algorithm generates the optimal broadcast time for a tree vertex in an FCT_n .

- Complexity = The same.



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- An optimal algorithm for broadcasting in an FCT_n ,
- Proof of correctness.
- Complexity: $O(|V| \log \log n)$,



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- The heuristic proposed for HT_k :
 - ◇ Extending the results for other networks: CCC_d , SE_d , BF_d , DE_d , etc.
 - ◇ Attaching random trees to the vertices of those networks,
 - ◇ Improving the heuristic itself:
 - When the heuristic fails?
When i wants to inform j , but $dist(i, j)$ is huge.
 - How to resolve it?
Consider a new label: $rem_{v,u}; v \in V_I, u \in V_U$.



Messy broadcasting

- Limited knowledge over the network,
 - ◇ Nodes must be able to act independently.

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- Limited knowledge over the network,
 - ◇ Nodes must be able to act independently.
- Three models:
 - ◇ M_1 : A vertex knows the state of its neighbors; informed-uninformed.
 - ◇ M_2 : A vertex knows from which vertices it received the message and to which vertices it has sent it.
 - ◇ M_3 : A vertex only knows to which vertices it has sent the message.



Messy broadcasting

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Conclusion and Timeline

- Limited knowledge over the network,
 - ◇ Nodes must be able to act independently.
- Three models:
 - ◇ M_1 : A vertex knows the state of its neighbors; informed-uninformed.
 - ◇ M_2 : A vertex knows from which vertices it received the message and to which vertices it has sent it.
 - ◇ M_3 : A vertex only knows to which vertices it has sent the message.
- Difference with classical model:
 - ◇ $t_i(u)$ is the maximum time units required to finish broadcasting from u , not minimum!



Messy broadcasting - cont.

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- Limited results so far [10]:
 - ◇ The exact values of $t_i(G)$ for K_n , P_n , C_n , and Complete d -ary trees for $i = 1, 2, 3$,
 - ◇ The exact values of $t_i(G)$ for H_k for only $i = 2, 3$,
 - ◇ Upper bounds for CCC_d , SE_d , BF_d , and DE_d for $i = 1, 2, 3$.



Messy broadcasting - cont.

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 - ◇ The exact values of $t_i(G)$ for K_n , P_n , C_n , and Complete d -ary trees for $i = 1, 2, 3$,
 - ◇ The exact values of $t_i(G)$ for H_k for only $i = 2, 3$,
 - ◇ Upper bounds for CCC_d , SE_d , BF_d , and DE_d for $i = 1, 2, 3$.
- Open problems:
 - ◇ $t_1(H_k)$,
 - ◇ Tightening the bounds for CCC_d , SE_d , BF_d , and DE_d .
 - ◇ Proposing a heuristic for general graphs.



Broadcasting using universal lists

- In classical model: Each node keeps a single list for each possible originator and need the complete knowledge over the network.

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Broadcasting using universal lists

- In classical model: Each node keeps a single list for each possible originator and need the complete knowledge over the network.
- Here, a universal list is maintained for all vertices.
- Two models:
 - ◇ Adaptive: Skip the ones you received from,
 - ◇ Non-adaptive: Skip none; send to all neighbors.

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Broadcasting using universal lists

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 - ◇ Adaptive: Skip the ones you received from,
 - ◇ Non-adaptive: Skip none; send to all neighbors.
- We want to introduce a third model:
 - ◇ Fully adaptive: Skip all informed vertices.

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Broadcasting using universal lists

- In classical model: Each node keeps a single list for each possible originator and need the complete knowledge over the network.
- Here, a universal list is maintained for all vertices.
- Two models:
 - ◊ Adaptive: Skip the ones you received from,
 - ◊ Non-adaptive: Skip none; send to all neighbors.
- We want to introduce a third model:
 - ◊ Fully adaptive: Skip all informed vertices.
- A trade-off between space complexity and the speed of broadcasting:

Model	No. of unnecessary calls	Space Complexity	Speed
Non-adaptive	Many	Very Low	Very Slow
• Adaptive	Few	Low	Slow
Fully Adaptive	0	Moderate	Moderate
Classical	0	Very high	Very Fast

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Broadcasting using universal lists - cont.

- Next step: Develop a comprehensive framework based on Genetic Algorithm for all three models using universal lists.

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Broadcasting using universal lists - cont.

- Next step: Develop a comprehensive framework based on Genetic Algorithm for all three models using universal lists.
- A candidate solution to the problem: A matrix $\sigma_{n \times \Delta(G)}$.
- Generate several random solutions,
- Using crossover and mutation over multiple generations, a relatively good solution could be found.

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Broadcasting using universal lists - cont.

- Next step: Develop a comprehensive framework based on Genetic Algorithm for all three models using universal lists.
- A candidate solution to the problem: A matrix $\sigma_{n \times \Delta(G)}$.
- Generate several random solutions,
- Using crossover and mutation over multiple generations, a relatively good solution could be found.
- Pros:
 - ◇ Works for arbitrary graphs,
 - ◇ Works for all three models,
 - ◇ Several fitness functions could be defined,
 - ◇ Efficient in terms of time complexity,
 - ◇ Gives the actual broadcast scheme,
 - ◇ The scheme could be used separately for proving many results,
 - ◇ ...

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- Considering broadcast problem,

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- Considering broadcast problem,
- So far:
 - ◇ A broadcasting heuristic for HT_k ,
 - *Published in [6]*
 - ◇ An optimal broadcast algorithm for FCT_n ,
 - *Submitted to [7]*

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Conclusion

- Considering broadcast problem,
- So far:
 - ◇ A broadcasting heuristic for HT_k ,
 - *Published in [6]*
 - ◇ An optimal broadcast algorithm for FCT_n ,
 - *Submitted to [7]*
- In the future:
 - ◇ Development for our heuristic,
 - ◇ Working on Messy broadcasting,
 - ◇ A new model based on universal lists,
 - ◇ A comprehensive framework using GA for broadcasting under universal lists.

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	T2	O2	
May	Jun.	Jul.	Aug.
T3	O3		
Sep.	Oct.	Nov.	Dec.
T4	O4		T5

2022			
Jan.	Feb.	Mar.	Apr.
O5		T6	O6
May.	Jun.	Jul.	Aug.
		T7	O7
Sep.	Oct.	Nov.	Dec.

No.	Description	Date	Done?	
T1	Task 1	Broadcasting heuristic in Hypercube of Trees	Nov. 2020	✓
O1	Outcome 1	Conference paper	Dec. 2020	✓ [24]
T2	Task 2	Optimal broadcasting in Fully Connected Trees	Feb. 2021	✓
O2	Outcome 2	Journal paper	Mar. 2021	✓ [25]
T3	Task 3	Introducing a new broadcast model for universal lists	May 2021	
O3	Outcome 3	Conference paper	Jun. 2021	
T4	Task 4	Broadcasting with universal lists with GA	Sep. 2021	
O4	Outcome 4	Journal paper + Ph.D. Seminar	Oct. 2021	
T5	Task 5	Messy broadcasting	Dec. 2021	
O5	Outcome 5	Journal paper	Jan. 2022	
T6	Task 6	Improved heuristic for Hypercube of Trees	Mar. 2022	
O6	Outcome 6	Journal paper	Apr. 2022	
T7	Task 7	Thesis writing	Jul. 2022	
O7	Outcome 7	Ph.D. defense	Aug. 2022	



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Thanks a bunch!