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BROADCAST GRAPHS WITH NODES OF LIMITED MEMORY

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Outline

- Introduction
- Background and Literature Review
- Comparison of universal list and messy broadcasting
- Broadcast graphs under the fully-adaptive model
- Conclusion and Future works

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Introduction

- Computer networks are becoming more popular each day!
- One problem: Propagate a message
- Information dissemination:
  - Unicasting,
  - **Broadcasting,**
  - Multicasting,
  - …

Introduction – cont.’

- Broadcasting is the process of distributing a message from a single node (originator) to all other nodes of the network,
- Each call is performed during one unit of time,
- Several calls could be performed in parallel,
- Three branches:
  - Classical broadcasting,
  - Broadcasting with Universal lists,
  - Messy broadcasting,

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Background & Literature Review

- Classical broadcasting $B_{cl}(G)$:
  - Consider vertex u as the originator,
  - Once a vertex v receives the message it will use its list l_v^u,
  - And pass the information to its uninformed neighbors following the order of its list.
  - The vital issue with the classical model:
    - The lists differ for various originators.
    - Each vertex must maintain several lists depending on the originator
  - A vertex must maintain $|V|$ different lists.

Background & Literature Review – cont.’

- Broadcasting with universal lists:
  - Every vertex of the network is given a universal list,
  - It has to follow the list, regardless of the originator.
  - when a vertex v receives the message, it should transmit the message to its neighbors following the ordering given in its list l_v.
  - Three sub-models:
    - *Non-adaptive* $B_{na}(G)$: skipping no vertex on the list,
    - *Adaptive* $B_a(G)$: skipping the vertices that a vertex received the message from that vertex,
    - *Fully-adaptive* $B_{fa}(G)$: skipping all informed neighbors.

Background & Literature Review – cont.'

- Broadcasting with universal lists:
  - Denote all possible schemes by $\sum$,
  - Model $M \in \{na, a, fa\}$,
  - For a scheme $\sigma \in \sum$: $B_M^\sigma(v, G)$: time steps needed to inform all the vertices in G from the source v while following the scheme σ under model M.
  - $B_M^\sigma(G) = \max_{v \in V}\{B_M^\sigma(v, G)\}$
  - $B_M(G) = \min_{\sigma \in \sum}\{B_M^\sigma(G)\}$

Background & Literature Review – cont.'

- Messy broadcasting:
  - every informed vertex randomly chooses a neighbor and sends the message.
  - Three sub-models:
    - *Model M_1:* once a vertex gets informed, it only has to send the message to its uninformed neighbors in some arbitrary order.
    - *Model M_2:* once vertex v gets informed, it will randomly send the message to the neighbors who have not sent it to v before.
    - *Model M_3:* a vertex will send the message to all of its neighbors in an arbitrary order once it gets informed.
  - The broadcast time of a vertex v under model M_i is denoted by $t_i(v)$ for $i = 1,2,3$:
    - maximum number of time units required to complete broadcasting originating from vertex v over all possible broadcast schemes.
  - $t_i(G) = \max_{v \in V}\{t_i(v)\}$

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Comparison of universal list and messy broadcasting

- **Lemma 1.** For any graph G:
  - $B_{fa}(G) \leq t_1(G)$,
  - $B_a(G) \leq t_2(G)$,
  - $B_{na}(G) \leq t_3(G)$,
- We also proved that these bounds are tight and cannot be improved in general.
- For any connected graph G with n vertices, m edges, diameter $d(G)$, and the maximum degree of Δ:
- **Corollary 1.** $B_{na}(G) \leq 2m - 1$ and $B_{fa}(G) \leq B_a(G) \leq m$.
- **Corollary 2.** $B_{na}(G) \leq d(G).\Delta$ and $B_{fa}(G) \leq B_a(G) \leq d(G).(\Delta - 1) + 1$

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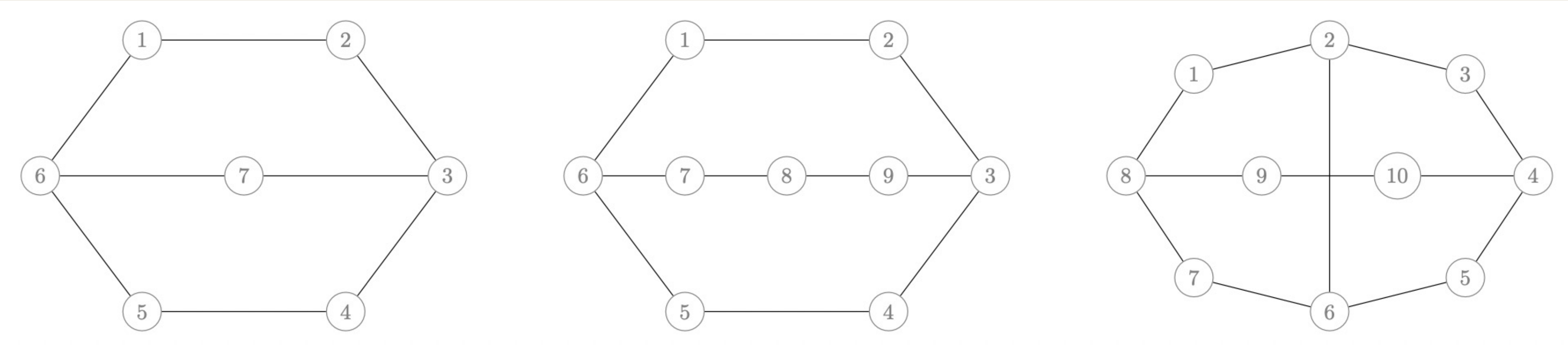
Broadcast graphs under the fully-adaptive model

- What is a *broadcast graph* (bg) ?
  - A graph $G = (V, E)$ in which broadcasting could be completed within the minimum possible time starting from any originator.
  - In classical model: minimum time = $\lceil \log n \rceil$, for a graph with n vertices.
    - $\forall u \in V(G): B_{cl}(u, G) = \lceil \log n \rceil$
- What is a *minimum broadcast graph* (mbg) ?
  - Is the bg with the minimum possible number of edges.
- $B^{(M)}(n)$: Broadcast function for an arbitrary value of n under model $M \in \{cl, fa\}$:
  - which is the number of edges associated with the mbg for n.
- Finding mbg's is very difficult and quite vital with a lot of applications,
- It has only been done for $M = cl$.

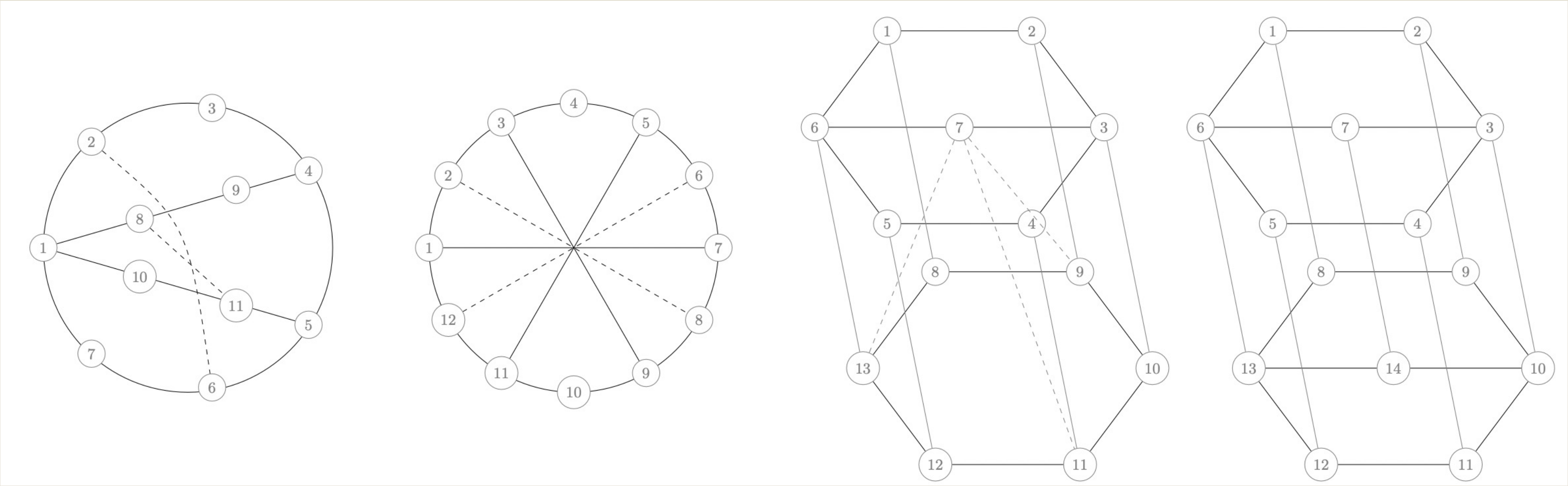
Broadcast graphs under the fully-adaptive model – cont.'

- **mbg's for some values of $n \leq 16$ under fully-adaptive model:**
- **Lemma 2.** If there is a graph G on n vertices for which $B_{fa}(G) = \lceil \log n \rceil$, then $B^{(cl)}(n) \leq B^{(fa)}(n)$.
- **Corollary 3.** Cycle C_n is mbg under fully-adaptive model for $4 \leq n \leq 6$.
- We also proved that the mbg's for classical broadcasting are also mbg's for the fully-adaptive model for $n = 7,9,10$.



Broadcast graphs under the fully-adaptive model – cont.'

- This problem becomes more difficult for $n \geq 11$ under the fully-adaptive model:
  - Our exhaustive search on more than 4000 broadcast schemes on mbg's with $n = 11, \ldots, 15$ vertices did not result in a broadcast scheme that achieves $B_{fa}^{\sigma}(G) = 4$.
- But we obtained an upper bound on the value of $B_{fa}(n)$ for $11 \leq n \leq 14$.



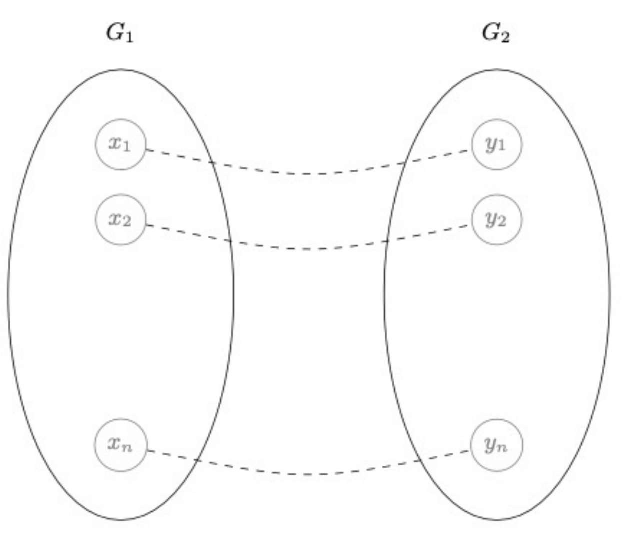
Broadcast graphs under the fully-adaptive model – cont.'

Table 1. The known values of $B^{(fa)}(n)$ for $n < 15$

n	3	4	5	6	7	8	9	10	11	12	13	14
Lower bound on $B^{(fa)}(n)$	2	4	5	6	8	12	10	12	13	15	18	21
Upper bound on $B^{(fa)}(n)$	2	4	5	6	8	12	10	12	15	17	23	23

Broadcast graphs under the fully-adaptive model – cont.'

- **General construction of bg's under fully-adaptive model:**
- We created first infinite families of broadcast graphs under the fully-adaptive model for $n = 6\times2^k, 7\times2^k, 9\times2^k, 10\times2^k$ for any value of $k \geq 1$.
- **Lemma 3.** Consider a graph $G = (V, E)$ with n vertices, m edges, and $B_{fa}(G) = \tau$.
  It is always possible to construct a graph $G' = (V', E')$ with $2n$ vertices, $2m + n$ edges, and $B_{fa}(G) = \tau + 1$.
- **Theorem 1.** $(G, n, m, \tau) \to (G', 2n, 2m + n, \tau + 1)$.
- **Corollary 5.** $(G, n, m, \lceil \log n \rceil) \to \left(G', 2^k.n, 2^k.m + k.2^{k-1}.n, \lceil \log n \rceil + k\right)$.



Broadcast graphs under the fully-adaptive model – cont.'

- **Theorem 2.** for any integer $k = \lceil \log n \rceil \geq 4$:
  - $B^{(fa)}(n) = B^{(fa)}(2^{k-1} + 2^{k-4}) \leq \frac{n\lceil \log n \rceil}{2} - \frac{8n}{9},$
  - $B^{(fa)}(n) = B^{(fa)}(2^{k-1} + 2^{k-3}) \leq \frac{n\lceil \log n \rceil}{2} - \frac{4n}{5},$
  - $B^{(fa)}(n) = B^{(fa)}(2^{k-1} + 2^{k-2}) \leq \frac{n\lceil \log n \rceil}{2} - \frac{n}{2},$
  - $B^{(fa)}(n) = B^{(fa)}(2^{k-1} + 2^{k-2} + 2^{k-3}) \leq \frac{n\lceil \log n \rceil}{2} - \frac{5n}{14},$
- **Theorem 3.** for a hypercube H_d: $B_{fa}(H_d) = d$
- **Corollary 6.** Hypercube H_d is an mbg on 2^d vertices under $M = fa$, and $B^{(fa)}(2^k) = k.2^{k-1}$ for any $k \geq 1$.

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Conclusion and Future works

- We proposed general upper bounds on the broadcast time of a graph G under the universal lists model,
- We studied broadcast graphs (bg)'s and minimum broadcast graphs (mbg)'s under the fully-adaptive model:
  - We presented mbg's on n vertices for $n \leq 10$,
  - And sparse bg's for $11 \leq n \leq 14$.
- Using our general construction, we suggested 4 infinite families of broadcast graphs under the fully-adaptive model,
- Lastly, we proved that Hypercube H_d is an $\mathrm{m}bg$ for any value of $n = 2^k$.

Conclusion and Future works- cont.'

- The problem of finding mbg's and bg's for greater values of n under universal lists models are still widely open,
- The broadcast time of several interconnection networks such as H_d are still unknown under adaptive and non-adaptive models.

THANK YOU!

Q/A?