A Broadcasting Heuristic for Hypercube of Trees

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Outline

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Introduction

- Computer networks are becoming more popular each day!
- One problem: Propagate a message
- Information dissemination:
  - Unicasting,
  - **Broadcasting**,
  - Multicasting,
  - ...

Introduction - cont.

- Broadcasting is the process of distributing a message from a single node (*originator*) to all other nodes of the network,
- Each *call* is performed during one unit of time,
- Several *calls* could be performed in parallel,
- We focus on broadcasting in a useful architecture, namely Hypercube of Trees

Outline

Problem Definition

- Given: $G(V, E)$, originator u,
- Calculate $b(u, G)$
  - NP-Complete in arbitrary graphs [5],
  - NP-Hard to approximate within a ratio of $(3 - \varepsilon)$ for any $\varepsilon > 0$ [3].
  - Solved optimally for only a few networks,
  - A long list of heuristics and approximation algorithms!

Problem Definition - cont.

- A Hypercube of Trees HT_k:
  - ◇ A hypercube of dimension k +
  - ◇ 2^k arbitrary trees.
- Very useful structure:
  - ◇ Distributing data [4],
  - ◇ Simultaneous exchange of packets between processors [1],
  - ◇ ...
- Current upperbound: $(2-\varepsilon)$-approximation [2]

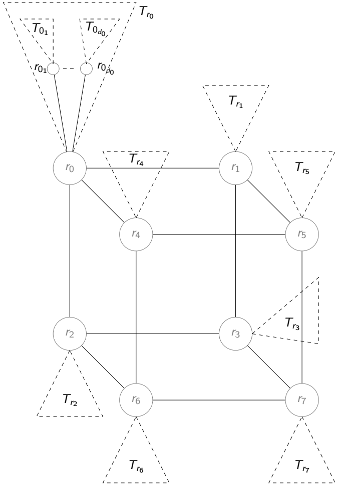

Figure: HT_3, A hypercube of trees with dimension 3

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- Do a binary search on this range (MBS).
  - Invoke the main heuristic (br) within this function.

Algorithm 1 The Modified Binary Search $MBS(G, r_i, lb, ub)$

input : $HT_k = (V, E)$, originator r_i, lower bound lb, and the upper bound ub
output: An improved broadcast time for $b(r_i, HT_k)$ denoted by b

```
b = ub
if ub < lb then
 |  return FALSE
end
mid = lb + ⌊(ub − lb)/2⌋
if br(HT_k, r_i, mid) then
 |  update b = mid
 |  return MBS(G, r_i, lb, mid − 1)
else
 |  return MBS(G, r_i, mid + 1, ub)
end
return b
```

Proposed Heuristic - *br*

- Considering a root vertex r_i at time t:
  - $b(r_i, T_i) > \tau - t$:
    - return FALSE!
  - $b(r_i, T_i) = \tau - t$:
    - r_i broadcasts in its tree T_i.
  - $b(r_i, T_i) < \tau - t$:
    - r_i must contribute in the Hypercube...

Proposed Heuristic - *br* - cont.

- We want to send a message from a root vertex r_i to another root vertex r_j
- Two questions must be answered:
  - How to choose r_j?
    - The one with the minimum value of rem_{r_j}:
      $rem_{r_j} = \tau - t - b(r_j, T_j) - dist(r_j, V_I)$

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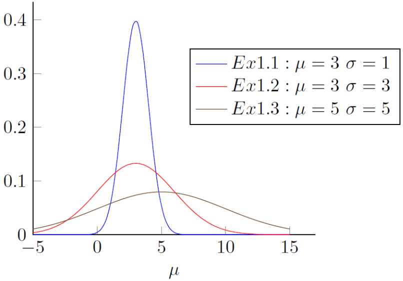
  - How to choose a path between r_i and r_j ?
    - ◇ A path P with the minimum value of c_P and d_P:
      $c_P = \min_{r_m \in P}\{rem_{r_m}\}$
      $d_P = \sum_{r_m \in P} rem_{r_m}$
    - ◇ More "critical" vertices on such a path!

Outline

Evaluation - Setup

- Generate 1000 random HT_k for each k.
  - ◇ Following Gaussian distribution $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$



- In terms of:
  - ◇ *success rate*: How many times we performed better than [2]?
  - ◇ *gain*: How much better?
  - ◇ $|V|$

Evaluation - Ex1.1

Table: Numerical Results for Ex1.1: $\mu = 3, \sigma = 1$

k	average $\|V\|$	success rate	average gain
3	115.86	29.79%	7.55%
4	233.55	28.49%	5.84%
5	466.09	21.3%	3.71%
6	932.49	5.7%	0.85%
7	1862.98	1.3%	0.16%
8	3729.59	0.2%	0.02%

Evaluation - Ex1.2

Table: Numerical Results for Ex1.2: $\mu = 3, \sigma = 3$

k	average $\|V\|$	success rate	average gain
3	614.28	73.5%	24.45%
4	1189.54	78.6%	23.31%
5	2481.77	74.5%	19.81%
6	5008.22	69.2%	15.55%
7	10003.63	63.2%	12.5%
8	19927.99	54.7%	9.76%

Evaluation - Ex1.3

Table: Numerical Results for Ex1.3: $\mu = 5, \sigma = 5$

k	average $\|V\|$	success rate	average gain
3	185255.48	81.20%	28.65%
4	280823.85	86.00%	28.82%
5	704372.23	88.10%	27.35%
6	1313690.28	89.30%	26.70%
7	3532669.06	90.50%	25.85%
8	5245921.21	90.10%	24.10%

Evaluation - Ex2.1

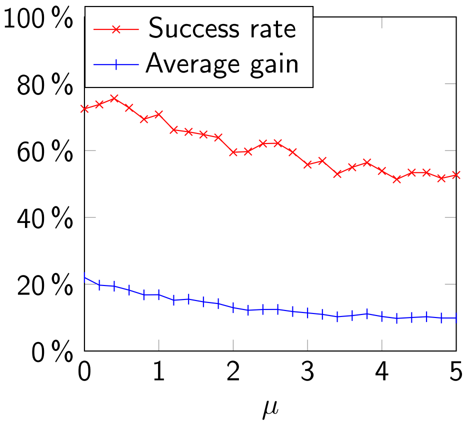


Figure: Ex2.1: $k = 5, \sigma = 2$

Evaluation - Ex2.2

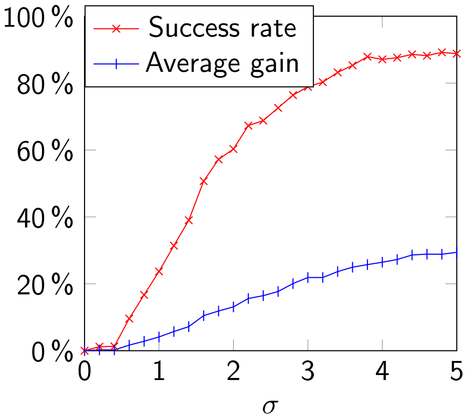


Figure: Ex2.2: $k = 5, \mu = 2$

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Conclusion

- We proposed a heuristic for broadcasting in a hypercube of trees.
- Theoretically:
  - 2-approximation
- Practically:
  - outperform the best-known algorithm for the same problem in up to 90% of the experiments while speeding up the process up to 30%

Future works

- Performance of the heuristic in real-world data sets,
- Approximability of this problem
  - Proposing additive approximation algorithm,
  - Or proving the NP-Completeness of the problem.
- Replace the hypercube with any other class of graphs
  - In which the broadcast scheme and broadcast time are known.

Important References I

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